Numerical Evidences for QED_3 being Scale-invariant

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Non-compact QED_3 on Euclidean ℓ^3 torus

Lagrangian

$$L = \overline{\psi}\sigma_\mu \left(\partial_\mu + iA_\mu\right)\psi + m\overline{\psi}\psi + \frac{1}{4g^2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2$$

- $\psi \rightarrow$ 2-component fermion field
- $g^2 \rightarrow$ coupling constant of dimension $[\text{mass}]^1$
  Scale setting $\Rightarrow g^2 = 1$
- massless Dirac operator: $C = \sigma_\mu \left(\partial_\mu + iA_\mu\right)$
  A special property for "Weyl fermions" in 3d: $C^\dagger = -C$
- Theoretical interests: UV complete, super-renormalizable and candidate for CFT
- Aside from field theoretic interest, QED_3 relevant to high-T_c cuprates.

Parity Anomaly and its cancellation

Parity: $x_\mu \to -x_\mu$

$$A_\mu \to -A_\mu; \qquad \psi \to \psi; \qquad \overline{\psi} \to -\overline{\psi}$$

$m\overline{\psi}\psi \to -m\overline{\psi}\psi \Rightarrow$ Mass term breaks parity (*i.e.*) the effective fermion action $\det C$ transforms as

$$\pm|\det C|e^{i\Gamma(m)} \to \pm|\det C|e^{i\Gamma(-m)} \overset{\text{reg}}{=} \pm|\det C|e^{-i\Gamma(m)}.$$

- When a gauge covariant regulator is used,

$$\Gamma(0) \neq 0 \qquad \text{(parity anomaly, which is Chern-Simons).}$$

- With 2-flavors of massless fermions, anomalies cancel when parity covariant regulator is used. We will only consider this case in this talk.

Parity and Gauge invariant regularization for even N

- Two flavors of two component fermions: ψ and χ.
- Define parity transformation: $\psi \leftrightarrow \chi$ and $\overline{\psi} \leftrightarrow -\overline{\chi}$.

Fermion action with 2-flavors

$$S_f = \begin{pmatrix} \overline{\psi} & \overline{\chi} \end{pmatrix} \begin{bmatrix} C+m & 0 \\ 0 & -(C+m)^\dagger \end{bmatrix} \begin{pmatrix} \psi \\ \chi \end{pmatrix}$$

- If the regulated Dirac operator for one flavor is C_{reg} and the other is $-C_{\mathrm{reg}}^\dagger$, theory with even fermion flavors is both parity and gauge invariant.
- Massless N-flavor theory has a $U(N)$ symmetry:

$$\begin{pmatrix} \psi \\ \chi \end{pmatrix} \rightarrow U \begin{pmatrix} \psi \\ \chi \end{pmatrix} \qquad U \in \mathrm{U}(2).$$

Mass explitly breaks $\mathrm{U}(N) \rightarrow \mathrm{U}\left(\frac{N}{2}\right) \times \mathrm{U}\left(\frac{N}{2}\right)$.

Parity and Gauge invariant regularization for even N

- Two flavors of two component fermions: ψ and χ.
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- If the regulated Dirac operator for one flavor is $C_{\rm reg}$ and the other is $-C_{\rm reg}^\dagger$, theory with even fermion flavors is both parity and gauge invariant.
- Massless N-flavor theory has a $U(N)$ symmetry:

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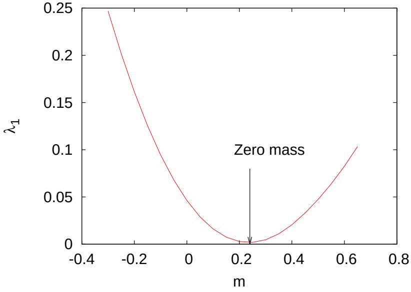
Mass explitly breaks $\mathrm{U}(N) \to \mathrm{U}\left(\frac{N}{2}\right) \times \mathrm{U}\left(\frac{N}{2}\right)$.

Parity-covariant Wilson fermions

Regulate one using $X = C_n - B + m$ and the other with $-X^\dagger = C_n + B - m$:

$$H_w = \begin{bmatrix} 0 & X(m) \\ X^\dagger(m) & 0 \end{bmatrix}$$

m $\rightarrow$ tune mass to zero as Wilson fermion has additive renormalization



Advantage: All even flavors N can be simulated without involving square-rooting.

Factorization of Overlap fermions

In 3d, the overlap operator for a single four component fermion (equivalent to $N = 2$) factorizes in terms of two component fermions:

$$H_{\text{ov}} = \begin{bmatrix} 0 & \frac{1}{2}(1+V) \\ \frac{1}{2}(1+V^\dagger) & 0 \end{bmatrix}; \qquad V = \frac{1}{\sqrt{XX^\dagger}} X$$

Advantages: All even flavors can be simulated without square-rooting; exactly massless fermions;

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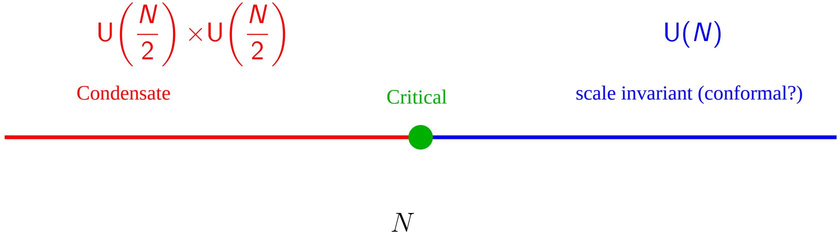
A few ways . . .

- Spontaneous breaking of U(N) flavor symmetry, leading to a plethora of low-energy scales like Σ, $f_\pi, \ldots$
- Particle content of the theory being massive
- Presence of typical length scale in the effective action: $V(x) \sim \log\left(\frac{x}{\Lambda}\right)$



Spontaneous breaking of U(N) symmetry

Large-N gap equation: $N_{\mathrm{crit}} \approx 8$ (Appelquist *et al.* '88)

Assumptions: $N \approx \infty$, no fermion wavefunction renormalization, and feedback from $\Sigma(p)$ in  is ignored.

Free energy argument: $N_{\mathrm{crit}} = 3$ (Appelquist *et al.* '99)

- Contribution to free energy: bosons$\rightarrow$ 1 and fermions$\rightarrow$ 3/2
- IR $\Rightarrow \dfrac{N^2}{2}$ Goldstone bosons + 1 photon
- UV $\Rightarrow$ 1 photon + N fermions
- Equate UV and IR free energies

Recent interest: Wilson-Fisher fixed point in $d = 4 - \epsilon$

Pietro *et al.*'15

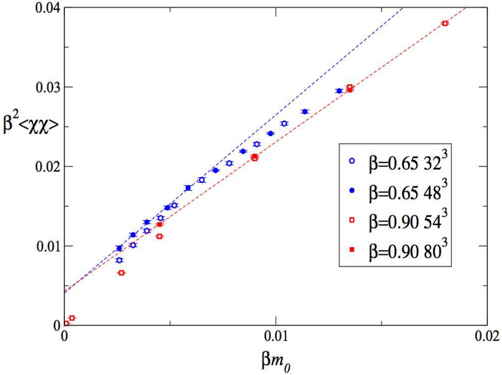
- IR Wilson-Fisher fixed point at $\frac{N g_*^2(\mu)}{\mu^\epsilon} = 6\pi^2\epsilon$
- Compute anomalous dimensions of four-fermi operators

$$O_\Gamma = \sum_{i,j} \overline{\psi_i}\Gamma\psi_i\overline{\psi_j}\Gamma\psi_j(x)$$

- *Extrapolate* to $\epsilon = 1$ and find O_Γ's become relevant at the IR fixed point when $N \approx$ 2-4.
- Caveats: mixing with $F_{\mu\nu}^2$ was ignored. Large-N calculation (Pufu *et al.*'16) seems to suggest that with this mixing, the dimension-4 operators remain irrelevant.

Previous attempts using Lattice

Hands *et al.*, '04 using square-rooted staggered fermions.



Condensate as a function of fermion mass.

Previous attempts using Lattice

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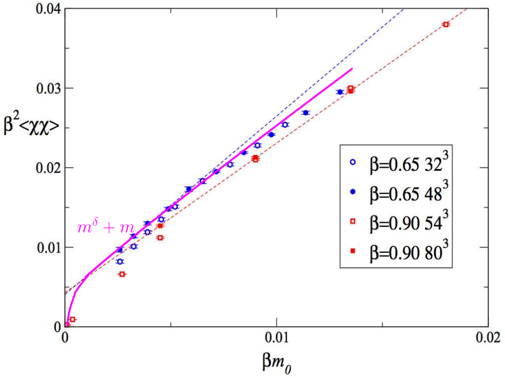

Method works if it is known a priori that condensate is present; A possible critical m^δ term, which would be dominant at small m, could be missed.

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Simulation details

Parameters

- L^3 lattice of physical volume ℓ^3
- Non-compact gauge-action with lattice coupling $\beta = \frac{2L}{\ell}$

Improved Dirac operator was used

- Smeared gauge-links used in Dirac operator
- Clover term to bring the tuned mass m closer to zero

Statistics

- Standard Hybrid Monte-Carlo
- 14 different ℓ from $\ell = 4$ to $\ell = 250$
- 4 different lattice spacings: $L = 16, 20, 24$ and 28
- $500 - 1000$ independent gauge-configurations

Computing bi-linear condensate from FSS of low-lying Dirac eigenvalues

(Wigner '55)

- Let a system with Hamiltonian H be chaotic at classical level.
- Let random matrix T, and H have same symmetries: UHU^{-1}
- Unfold the eigenvalues *i.e.*, transform $\lambda \to \lambda^{(u)}$ such that density of eigenvalues is uniform.

$$\lambda^{(u)} = \int_0^{\lambda} \rho(\lambda) d\lambda$$

- The combined probablity distribution $P(\lambda_1^{(u)}, \lambda_2^{(u)}, \ldots)$ is expected to be universal and the same as that of the eigenvalues of T

Computing bi-linear condensate from FSS of low-lying Dirac eigenvalues

- Banks-Casher relation$\Rightarrow$ non-vanishing density at $\lambda = 0$

$$\Sigma = \frac{\pi\rho(0)}{\ell^3}; \qquad \text{where} \qquad \int_0^\infty \rho(\lambda) d\lambda = \ell^3$$

- Unfolding $\Rightarrow \lambda^{(u)} \approx \rho(0)\lambda \sim \Sigma\ell^3\lambda$. Therefore, universal features are expected to be seen in the microscopic variable z:

$$z = \lambda\ell^3\Sigma.$$

- $P(z_1, z_2, \ldots, z_{\max})$ is universal and reproduced by random T with the same symmetries as that of Dirac operator D. (Shuryak and Verbaarschot '93)
- Rationale: Reproduces the Leutwyler-Smilga sum rules from the zero modes of Chiral Lagrangian.
- Eigenvalues for which agreement with RMT is expected / Momentum scale upto which only the fluctuations of zero-mode of Chiral Lagrangian matters:

$$z_{\max} < F_\pi \ell \qquad \text{(Thouless energy)}$$

RMT and Broken phase: Salient points

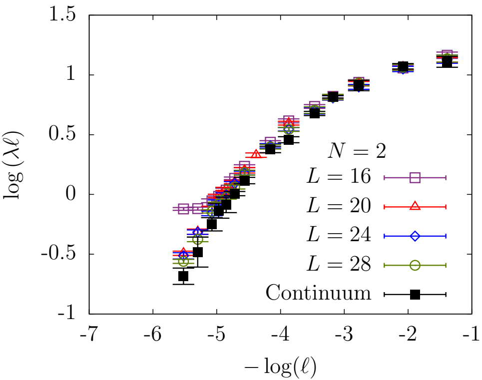
- Scaling of eigenvalues:

$$\lambda\ell \sim \ell^{-2}$$

- Look at ratios $\lambda_i/\lambda_j = z_i/z_j$. Agreement with RMT has to be seen without any scaling.

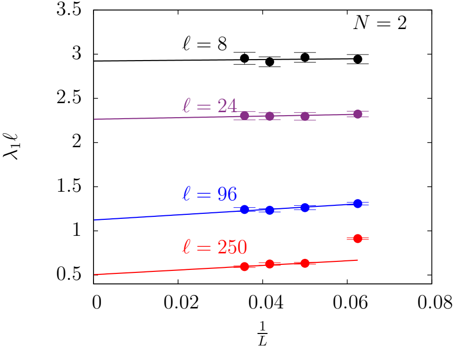
- The number of microscopic eigenvalues with agreement with RMT has to increase linearly with ℓ

Finite size scaling of eigenvalues: continuum limits

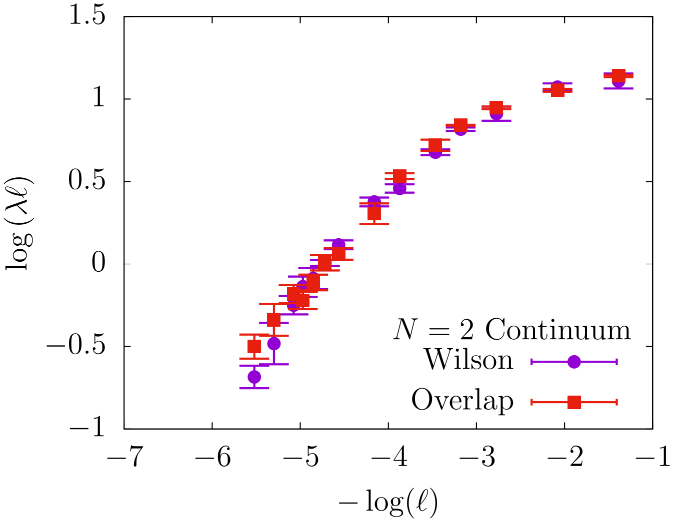


Lattice spacing effect using Wilson fermions

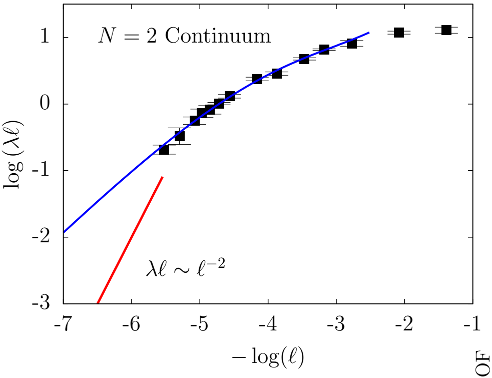
Find continuum limit at each fixed ℓ.



Agreement between Wilson and Overlap



Absence of bi-linear condensate: $\lambda \sim \ell^{-1-p}$ and $p \neq 2$

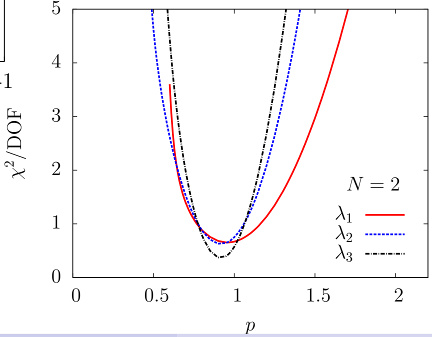


- $\lambda\ell \propto \ell^{-1}$ seems to be prefered.
- The condensate scenario, $\lambda\ell \propto \ell^{-2}$ seems to be ruled out.

Ansatz:

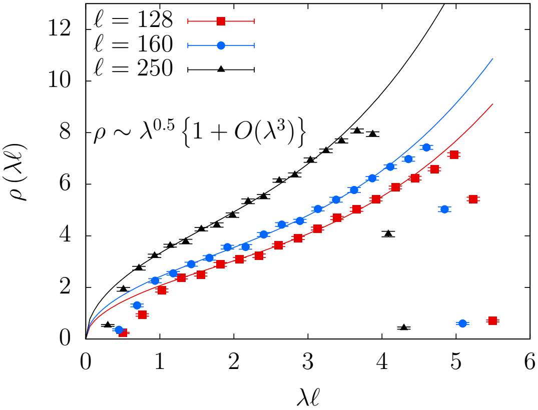
$$\log(\lambda\ell) = \frac{a - (p + \frac{b}{\ell}) \log(\ell)}{1 + \frac{c}{\ell}}$$

Robustness: Changing ansatz to $\lambda\ell \sim \ell^{-p}\left(1 + \frac{a}{\ell} + \ldots\right)$ changes the likely p from 1 to 0.8.



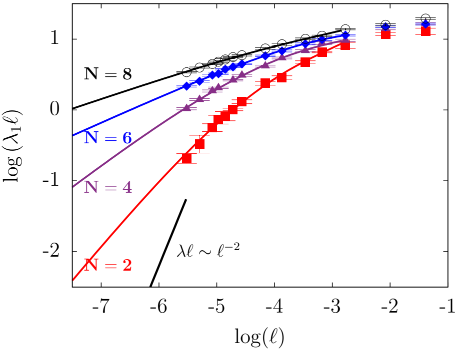
Eigenvalue density

$\lambda \sim \ell^{-1-p} \Rightarrow \quad \rho(\lambda) \sim \lambda^{(2-p)/(1+p)}$ and $\Sigma(m) \sim m^{(2-p)/(1+p)}$ DeGrand '09



$\rho \sim \lambda^{0.5}$ in the bulk

$N = 2, 4, 6, 8$



Ansatz:

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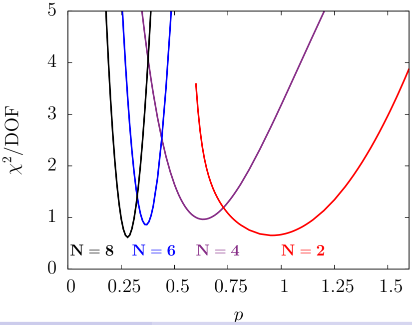


- p decreases with N: trend$\Rightarrow$ $p \approx \frac{2}{N}$
- $p \approx 1$ is right at the edge of allowed value from CFT constraints.

Absence of condensate using Inverse Participation Ratio

- For normalized eigenvectors of D

$$I_2 = \int |\psi(x)|^4 d^3x$$

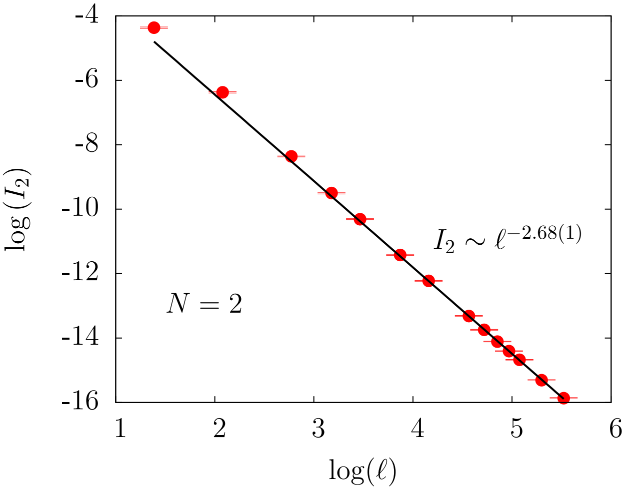
- Volume scaling

$$I_2 \propto \ell^{-(3-\eta)}$$

- Condensate $\Rightarrow$ RMT $\rightarrow \eta = 0$.
- Localized eigenvectors $\rightarrow \eta = 3$.
- Eigenvector is multi-fractal for other values.

A multifractal IPR

A theory with condensate is analogous to a metal. Multifractality is typical at a metal-insulator critical point.



Spectrum of massless QED$_3$

$$\text{“Pion” :} \qquad O_\pi(x) = \overline{\psi}\chi(x) \pm \overline{\chi}\psi(x)$$

$$\text{“Rho” :} \qquad O_\rho(x) = \overline{\psi}\sigma_i\chi(x) \pm \overline{\chi}\sigma_i\psi(x)$$

Theory with a scale:

$$\left\langle O(x)O(0)\right\rangle \sim \exp\{-Mx\}$$

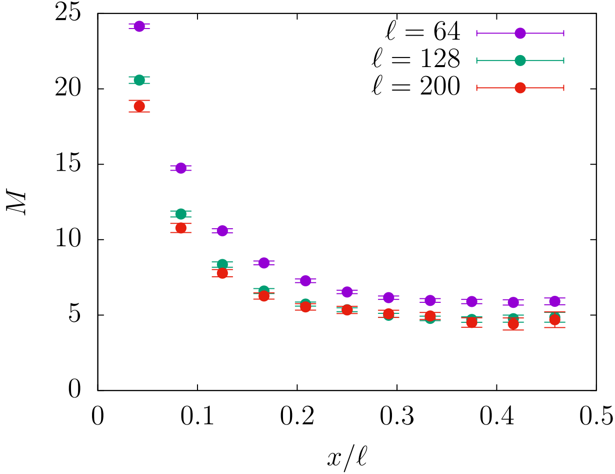
Scale-invariant theory:

$$\left\langle O(x)O(0)\right\rangle \sim \frac{1}{|x|^\delta} f\left(\frac{x}{\ell}\right) \longrightarrow \frac{1}{|x|^\delta}\exp\left\{-M\frac{x}{\ell}\right\}$$

Extract M by fits to correlators. To extract δ, one needs both ℓ large and $Mx \ll \ell$. We do not have control over both scales.

Spectrum of massless QED$_3$

Effective mass shows a plateau as a function of x/ℓ— Scaling function is $\exp\left\{-M\frac{x}{\ell}\right\}$

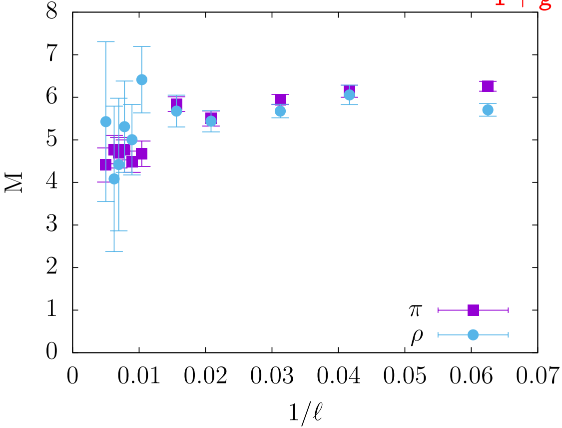


Spectrum of massless QED_3

As $\ell \to \infty$, M has a finite limit for both π and ρ.

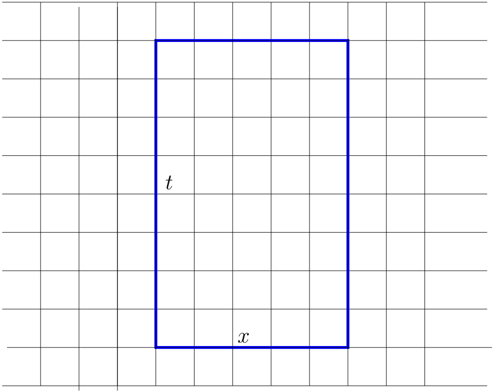
The plateau in M as a function of ℓ could imply the vanishing of $\beta = \dfrac{dg_R^2(\ell)\ell}{d\log\ell}$

near the IR fixed point as $\ell \to \infty$ (*i.e.*) if $M \propto g_R^2(\ell)\ell = \dfrac{\#g^2\ell}{1+g^2\ell} \to \#$



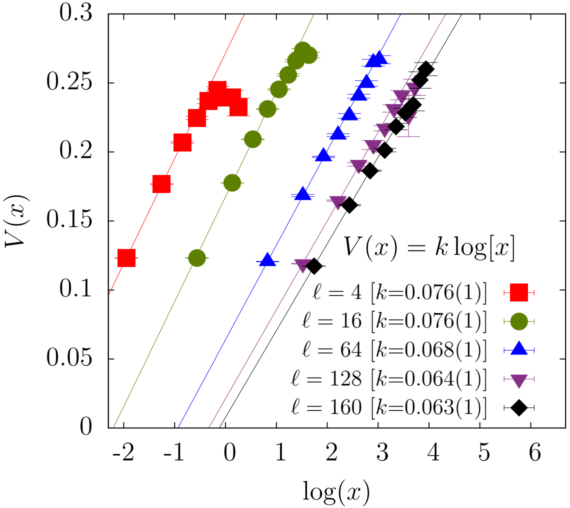
Absence of scale in $\log(x)$ potential

$t \times x$ Wilson loop $\rightarrow \log(\mathcal{W}) = A + V(x)t$



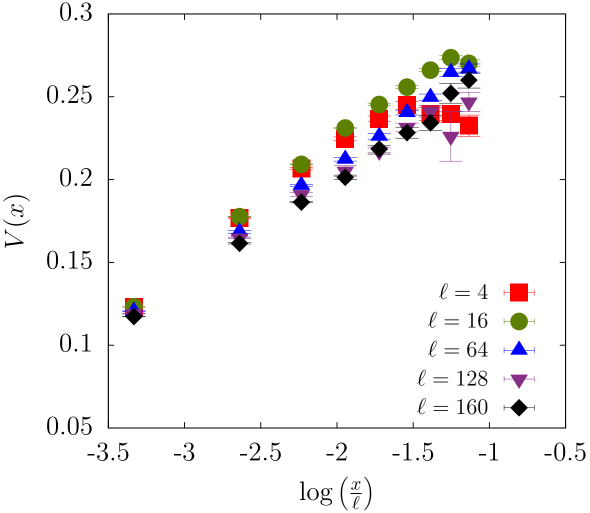
Absence of scale in $\log(x)$ potential

If $V(x) \sim \log\left(\frac{x}{\Lambda}\right)$, it would have a well defined limit at fixed x when $\ell \to \infty$



Absence of scale in $\log(x)$ potential

Instead, a scale invariant potential $V(x) \sim \log\left(\frac{x}{\ell}\right)$



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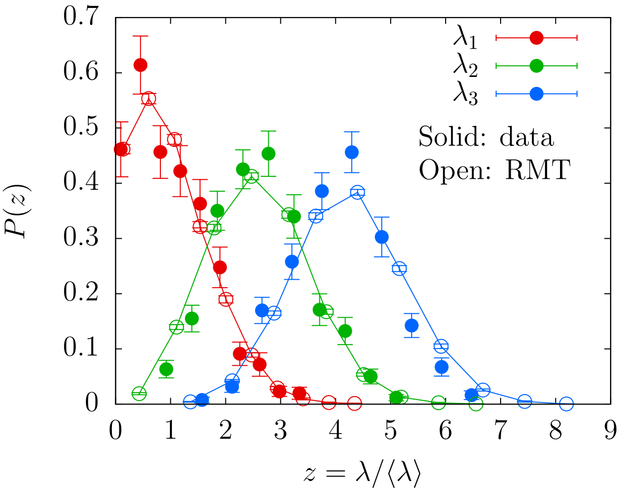
Finding bilinear condensate in large N_c in 3d

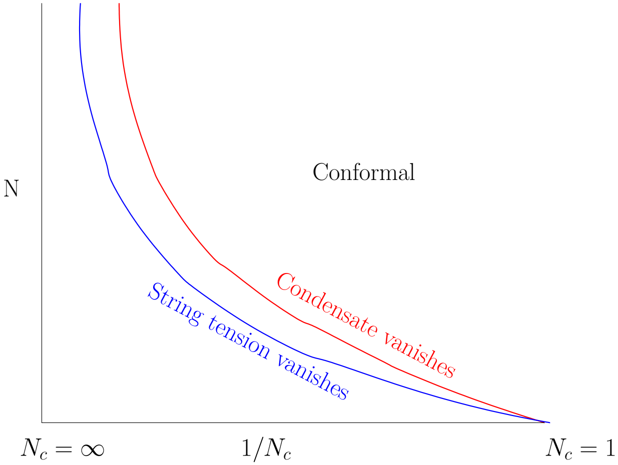
- Pure non-abelian gauge theories in 3d have string tension. Questions: With N flavors of fermions, do they have bilinear condensate? Critical N (or different critical N's) at each N_c where condensate and string tension vanish?
- First step: Large N_c, where quenched approximation is exact.
- Assume partial volume reduction for $\frac{1}{\ell} < T_c$. We keep the lattice coupling $\beta < \beta_c$ on 5^3 lattice with $N_c = 7, 11, \ldots, 37$. Determine the eigenvalues of the Hermitian overlap operator.

Agreement with Non-chiral RMT

Quenched $\Rightarrow$ $$Z_{\mathrm{RMT}} = \int e^{-\mathrm{Tr}\,T^2} dT \qquad ; T = T^\dagger$$



A guess



Conformal
N
Condensate vanishes
String tension vanishes
$N_c = \infty$
$1/N_c$
$N_c = 1$

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Conclusions